Kirchhoff depth migration using maximum amplitude traveltimes computed by the Chebyshev polynomial recursion

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This paper was prepared for presentation at the 14th International Congress of the Brazilian Geophysical Society, held in Rio de Janeiro, Brazil, August 03-06, 2015.

Abstract

We describe in this paper a new procedure for computing traveltimes for a grid of points by solving the wave equation using the rapid expansion method (REM). Using the REM approach, the wave equation solution is expanded in a series of Chebyshev polynomials. The Chebyshev polynomials can tell us where the wave will go but not when. Their wavelike character result from the repeated recursive application of the Laplacian that generates the Chebyshev polynomials. Thus, in order to identify the arrival time on each grid point, we use the maximum amplitude criterion and the arrival time on each point of the grid is assigned by taking the correspondent Chebyshev recursion number related to the maximum amplitude event.

To demonstrate the efficiency and applicability of our alternative procedure to calculate traveltimes, using the Chebyshev polynomial recursion, we apply it to the Kirchhoff operator for depth migration. Next, the migration results from two synthetics dataset using traveltime tables computed by a conventional Kirchhoff migration, using ray tracing traveltime algorithm, are compared with the Kirchhoff migration results, using traveltimes computed by the chebyshev polynomial, recursion combined with the maximum amplitude criterion.

Introduction

The Kirchhoff operator is currently the most widely used migration operator, because of its computational efficiency, steep dip accuracy and potential I/O flexibility. The common method used to compute Green's function for Kirchhoff migration is based on the high-frequency ray approximation. This approximation leads to traveltimes independent of the amplitude, where the ray path and consequently the traveltimes are found by solving the eikonal equation and amplitudes are solved with the transport equation (Aki and Richards, 1980).

In most of the cases, traveltime tables for Kirchhoff operator are calculated using first arrivals by ray-tracing technique employing as input a smooth version of the velocity model, i.e. some information in the the velocity model is lost and the classical ray shooting that propagates single rays through the medium with complex geology usually leads to poor illumination.

In general, the Kirchhoff migration operator is implemented using traveltimes computed by a ray tracing algorithm and it often does not work well in complex media.

In the literature, the traveltimes tables are computed using different techniques to improve the Kirchhoff operator performance for migration. These can be summarized as: single-arrival Kirchhoff prestack depth migration using a full-wave equation solution, maximum amplitude arrival traveltime, band-limited Green's function, paraxial maximum-amplitude arrival (Audebert et al., 1997).

Traveltimes can also be computed by methods which are based on the solution of the full wave equation. Finite difference (FD) is a well known and popular numerical solution for the wave equation. It has been common to use FD approximation for both time and spatial evolution of wavefields. Although easy to solve, it is only conditionally stable, which imposes a limit on the marching time step size. In addition to that, all the finite difference methods suffer from numerical dispersion problems. Various alternative approaches have been proposed in the geophysical literature to achieve stability and dispersion-free extrapolation of scalar waves in heterogeneous media for large time steps (Du et al., 2014).

Among these methods, we have the rapid expansion method (REM), which makes use of the Chebyshev expansion (Kosloff et al., 1989; Pestana and Stoffa, 2010). This method has very high accuracy with respect to the temporal extrapolation. The combination of the REM approach and the Fourier method allows one to obtain modeling results that are free of numerical dispersion.

In the paper presented by Pestana and Stoffa (2010), a numerical example of seismic modeling for a salt-model, shows the plot of several Chebyshev polynomials. Even though the response of the Chebyshev polynomials appears as propagating waves, time is not known for each plot. We can also notice that the Chebyshev polynomial results can tell us where the waves will go but not when. Moreover, in these plots we can see waves propagating through the medium and their wavelikes character results from the repeated recursive application of the Laplacian that generates the Chebyshev polynomials.

Based on an experiment presented in Pestana and Stoffa (2010), we are proposing a simple and alternative way to compute traveltimes. The proposed procedure utilizes the Chebyshev recursion from the one-step solution of the acoustic wave equation by REM. In order to identify the arrival time on each grid point, we use the maximum amplitude criterion and the arrival time for each point of the grid is assigned be taking the Chebyshev recursion number related to the maximum amplitude of the wavefield on each grid point of the model.

To demonstrate the superiority of the Kirchhoff migration using the proposed traveltime computation, we show the results using two synthetic datasets with complex geological structures. We have tested our Kirchhoff migration algorithm for the SEG/EAGE and Marmousi models. The migration results are compared with the results obtained through the Kirchhoff migration using traveltimes computed using the Seismic Unix (Stockwell, 1999), where the traveltime tables are calculated by paraxial ray tracing and traveltimes from the shadow zones are replaced by the traveltimes obtained by solving the eikonal equation.

Theory

Rapid expansion method: a one step solution

First presented by Kosloff et al. (1989), the rapid expansion method (REM) solves the acoustic wave equation with zero initial conditions. The acoustic wave equation is written in operator notation as:

$$\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2} = -L^2 P(\mathbf{x},t) + S(\mathbf{x},t), \tag{1}$$

where $P(\mathbf{x},t)$ denotes the pressure wavefield, $S(\mathbf{x},t)$ is the source term, $\mathbf{x} = (x,y,z)$, with x, y and z Cartesian coordinates and t is time.

For a 3-D constant density acoustic wave equation, $-L^2$ is given by:

$$-L^2 = c^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right), \tag{2}$$

where c is the velocity.

The formal solution to equation 1 with initial condition can be found as (Kosloff et al., 1989; Carcione and Seriani, 1992)

$$P(\mathbf{x},t) = \int_0^t \frac{sen\left[L(t-\tau)\right]}{L} S(\mathbf{x},\tau)\,\mathrm{d}\tau. \tag{3}$$

If the source term $S(\mathbf{x},t)$ is separable, i.e.,

$$S(\mathbf{x},t) = g(\mathbf{x}).h(t), \tag{4}$$

where all time dependence is contained in $h(t)$, the solution 3 can be written as

$$P(\mathbf{x},t) = \mathbf{G} * h(t) \tag{5}$$

and

$$\mathbf{G} = \frac{sen(Lt)}{L} g(\mathbf{x}), \tag{6}$$

where $*$ denotes convolution in time.

Now, using a modified Chebyshev polynomial expansion to the sine function, which gives (Kosloff et al., 1989; Carcione and Seriani, 1992)

$$P(\mathbf{x},t) = \sum_{k=0}^{\infty} b_{2k+1}(t) \left(\frac{R}{iL}\right) Q_{2k+1}\left(\frac{iL}{R}\right) g(\mathbf{x}) \,, \tag{7}$$

and

$$b_{2k+1}(t) = \frac{1}{R} \int_0^t J_{2k+1}(\tau R)\, h(t-\tau)\,\mathrm{d}\tau, \tag{8}$$

where J_k is the Bessel function of order k, Q_k are the modified Chebyshev polynomials and for $2D$ wave propagation, the value of R is given approximately by: $R = \pi c_{max} \sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}$, where c_{max} is the maximum velocity in the mesh and Δx and Δz are the grid spacings in x- and z- directions, respectively.

In expression 7 we need to make sure that the number of terms used by the Chebyshev expansion converges, providing a good approximation of the sine function such that the wavefield extrapolation can occur in a stable form. The REM converges exponentially if $k > tR$. The summation can be safely truncate with a k_{max} value slightly greater the tR (Tal-Ezer, 1986)

Due to the fact that the solution of equation 1 only contains odd polynomials, it is more convenient to use the following recursion relation

$$Q_{2k+1}(x) = 2\left[1 + 2x^2\right] Q_{2k-1}(x) - Q_{2k-3}\,(x), \tag{9}$$

To initiate the recurrence, we need the terms:

$$Q_1(x) = x, \quad \text{and} \quad Q_3(x) = 3\,x + 4\,x^3, \tag{10}$$

where we have replaced x by $\frac{iL}{R}$.

The Kirchhoff operator

This work considers a depth migration procedure using a Kirchhoff operator. Kirchhoff migration involves integrating traces amplitudes over a reflectivity model. After traveltimes and Kirchhoff weights have been calculated, the migration process can be written as a trace by trace process.

Given the simplicity of the Kirchhoff adjoint operator, the forward operator is straightforward to define.

The 2-D forward Kirchhoff operator can be written as

$$d(x_s,x_g,t) = \sum_{N_x}\sum_{N_z} m(z,x) K(x_s,x_g,x,z,t), \tag{11}$$

where $d(x_s,x_g,t)$ is the data, $m(x,z)$ is the model, and $K(x_s,x_g,x,z,t)$ are the Kirchhoff weights. Here x_s and x_g are the source and receiver position, respectively, and t is the traveltime that is normally obtained via ray tracing through the velocity model.

The adjoint, or migration operator, can be written as

$$m(z,x) = \sum_{N_s,N_z} d(x_s,x_g,t) K(x_s,x_g,x,z,t). \tag{12}$$

Given the forward Kirchhoff operator, the data can be generated from the reflectivity model. Given recorded data, we may want to collapse diffractions to the position where reflections occurred.

Maximum amplitude traveltime by Chebyshev polynomial recursion

Computation of the travel times is the heart of the Kirchhoff algorithm. Ray tracing is the most used method to compute the arrival times. An alternative procedure for computing the arrival times for a grid of points is by solving the eikonal equation by finite-difference method.

Here we suggest an alternative procedure to calculate the arrival times based on the maximum amplitude criterion. The solution of the wavefield response for an injected point source based on the REM (Kosloff et al., 1989) is given by equation 7 for a predefined propagation time t. Using the Chebyshev recurrence relation (expression 9), we can compute all Chebyshev polynomials based on the series convergence, i.e., k_{max} has to be greater then tR (Tal-Ezer, 1986).

To determine the traveltime from the Chebyshev polynomials we can use the maximum amplitude criterion and thus we can identify the direct wave computed from the modeling. This maximum amplitude criterion is justified since the direct wave has the maximum amplitude at the direct arrival time. Late arrivals have smaller amplitudes due to the transmission losses. Using the maximum amplitude criterion the travel time $t_{i,j}$ for each grid point (i,j) at each k-step of the Chebyshev recursion is updated and after finishing the last step of the Chebyshev recursion a traveltime table is saved in a file to be used as input for the Kirchhoff migration and modeling procedures.

Based on the following relation and using the maximum amplitude of the Chebyshev polynomial "wavefield" , we can assign the traveltime on each grid point of the model, which is given by:

$$t = \frac{2k}{R}, \tag{13}$$

where k is the number of the Chebyshev recursion and R is as defined above.

NUMERICAL RESULTS

Poststack migration results

To demonstrate the efficiency and applicability of the proposed algorithm to compute traveltimes, we apply Kirchhoff migration and compare the migration results in terms of imaging quality using the traveltime tables computed by paraxial ray tracing (SU code) with the migration results of Kirchhoff migration using traveltimes obtained by Chebyshev recursion, considering the maximum amplitude criterion.

The first example taken for testing is the synthetic SEG/EAGE dataset. It is a very well tested zero-offset dataset sampled in time at $\Delta t = 0.008\,s$ and with $nt = 626$ samples per trace. The SEG/EAGE velocity model is shown in Figure 1. This velocity model is sampled spatially with $\Delta x = \Delta z = 0.012\,km$. the model has 1290 points in the horizontal direction (x) with 300 in the vertical direction (z).

Figure 2 shows the zero-offset migration result using Kirchhoff migration algorithm with the traveltime computed by Chebyshev recursion and maximum amplitude criterion. In this case, the true velocity model is employed to compute the traveltimes as described above. To compute the traveltime based on the first arrival ray tracing algorithm, we need to have a smoothed version of the velocity model. Thus, to compute the ray tracing traveltimes we used the smoothed velocity model for the SEG/EAGE shown in Figure 3 and the Kirchhoff migration result using the ray tracing first arrival traveltime presented in Figure 4.

Comparing the Kirchhoff migration results for the postack migration of the SEG/EAGE dataset, we can notice a superior migrated imaging using the Chebyshev traveltimes. We can also see that the Chebyshev traveltime with maximum amplitude criterion showed a better imaging of the reflectors below the salt and also produced a better image of the base and top of the salt body.

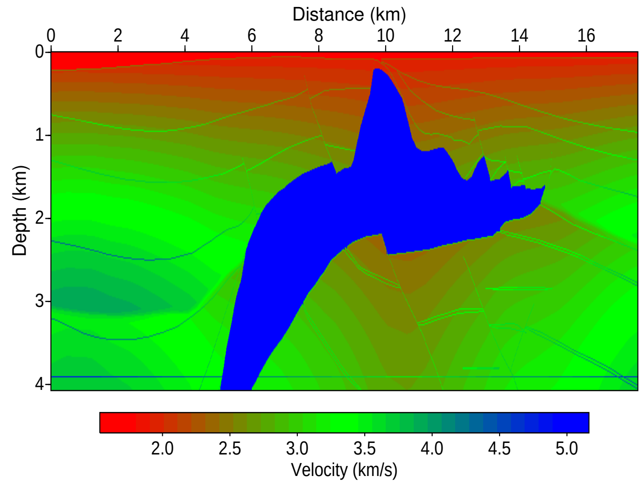

Figure 1: SEG/EAGE salt velocity model

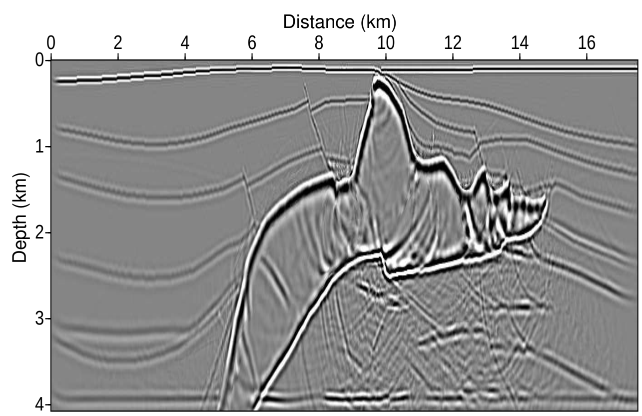

Figure 2: SEG/EAGE Kirchhoff migration result obtained based on the maximum amplitude traveltimes computed by Chebyshev recursion and using the true velocity model (Figure 1)

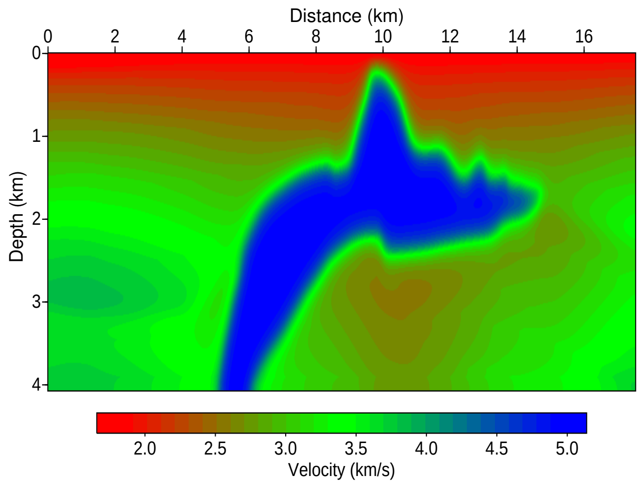

Figure 3: SEG/EAGE salt smoothed velocity model

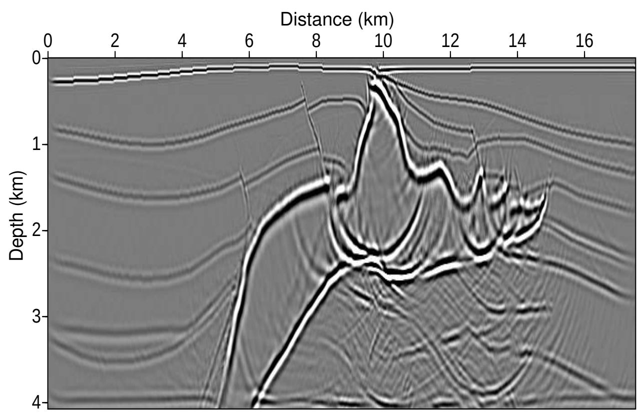

Figure 4: SEG/EAGE Kirchhoff migration result obtained using ray tracing first arrival traveltimes using the SEG/EAGE smoothed velocity model (Figure 3)

Prestack migration results

We also tested the traveltime algorithm proposed here on the Marmousi dataset. The Marmousi is constructed based on a profile of actual geology, the velocity model and structure are very complicated, and since then it has become a popular test dataset for advanced migration methods (Audebert et al., 1997; Bevc, 1995). The velocity model, showed in Figure 5, has 369 points in the horizontal direction (x) and 375 in the vertical direction (z) and the spacings are $\Delta x = 25$m and $\Delta z = 8$m. In the velocity model the velocities vary from $1500\ m/s$ to $5500/,m/s$. In Figure 5 we also show some traveltime curves computed by maximum amplitude criterion using the Chebyshev recursion overlaid the Marmousi velocity model. The Marmousi dataset used here has 240 shots and each shot gather has 96 traces and each trace has 725 samples with a time sampling interval of $4ms$ with $nt = 725$.

Figure 7 shows a smoothed version of velocity model needed by the ray tracing algorithm to be used to compute the arrival traveltimes.

Figures 6 and 8 show the Kirchhoff migration results generated from traveltimes by maximum amplitude Chebyshev recursion and by ray tracing traveltimes, respectively. For the maximum amplitude traveltime Chebyshev recursion algorithm we employed the Marmousi true velocity model (Figure 5) and for the ray tracing algorithm we used its smoothed version (Figure 7).

Again, using this prestack dataset, we can see that the Kirchhoff migration result using the traveltimes proposed here shows a better image than one obtained by the the Kirchhoff migration using ray tracing traveltimes. Moreover, the result after applying the Laplacian filter to remove the low frequency noise (Figure 9) has much better S/N ratio and shows much better imaging, especially in the lower part of the model in comparison with the ray tracing traveltimes migration result. So far, we have from these results that the Kirchhoff migration using the traveltime procedure proposed here can produce better results than the traditional Kirchhoff migration method with ray tracing traveltimes, especially in areas with complex geology, such as those affected by salt tectonics.

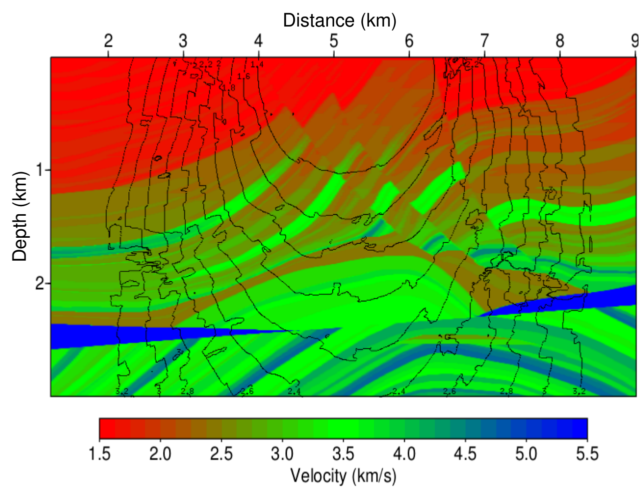

Figure 5: Marmousi velocity model on background and on the top of it maximum amplitude traveltime curves computed by Chebbyshev recursion for a source at 4120 m and a receiver at 6040 m.

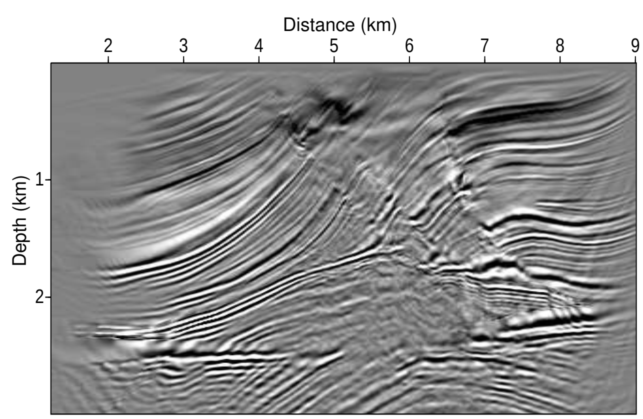

Figure 6: Kirchhoff migration result using maximum amplitude traveltime computed by Chebyshev recursion

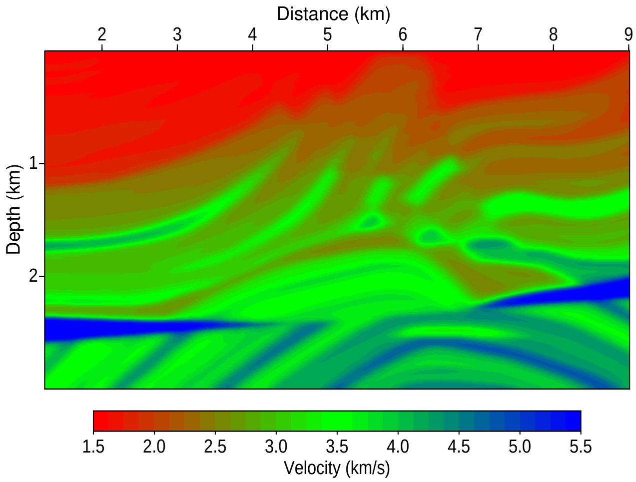

Figure 7: Smoothed version of the Marmousi velocity model

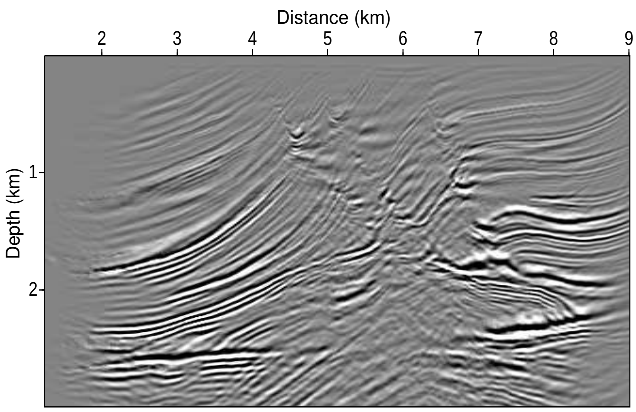

Figure 8: Kirchhoff migration result using ray tracing traveltimes. The traveltimes were computed using the smoothed version of the Marmousi velocity model.

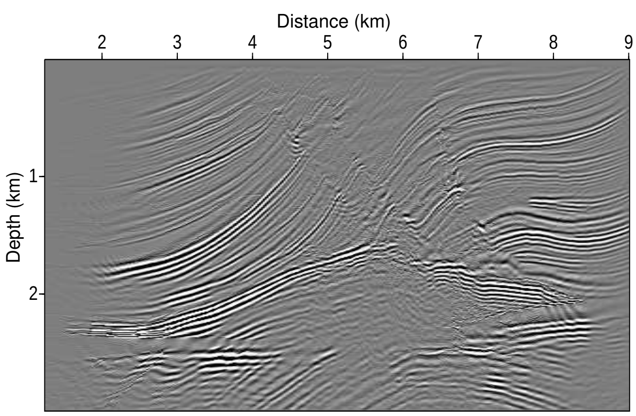

Figure 9: Kirchhoff migration result using maximum amplitude traveltimes by the Chebyshev recursion, after the Laplacian filter to remove the low frequency noise

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Conclusions

We have proposed an alternative and simple procedure to calculate traveltimes using the REM wave equation solution. Here the traveltimes are computed using the maximum amplitude criterion applied for the Chebyshev polynomials ”wavefield”. As the REM is a very accurate method and its combination with the Fourier method allows one to obtain modeling results that are free of numerical noise, it can used to generate much better traveltime tables for Kirchhoff migration compared with traveltimes based on the full wave-equation solution.

From the Kirchhoff migration results, for both the EAGE/SEG and Marmousi models, which are complex geological structures, we showed that the migrated sections obtained here using the traveltime computed by the proposed algorithm, showed very goods results when compared with the Kirchhoff migration results using ray tracing traveltimes.

We conclude that the alternative procedure for traveltime computation based on the maximum amplitude by the Chebyshev recursion can be applied with the Kirchhoff migration operator to produce accurate results especially for complex geology, as showed in our results.

Acknowledgements

The authors thank CPGG/UFBA, CNPq and INCT-GP/CNPq for supporting the development of this work.

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